# UNCERTAINTY VISUALIZATION

# using

# High Dynamic Range Volume Rendering

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**Master of Engineering**
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by

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TO

*my parents*

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# Abstract

*We propose a method for uncertainty visualization of scalar volumetric data sets using high dynamic range (HDR) volume rendering. We design a transfer function that maps each data point to a color in HDR space. We modify existing tone mapping techniques and suitably integrate them with direct volume rendering to obtain a low dynamic range image that is rendered on screen. The resulting visualization allows the user to study the details in data as well as uncertainty.*

# Contents

# List of Figures

# Keywords

Uncertainty visualization
Transfer function design
Ray casting
Volume rendering
High dynamic range imaging
Tone mapping

# Notation and Abbreviations

### Notations

No notations have been used in this document.

### Abbreviations

HDR : High Dynamic Range

LDR : Low Dynamic Range

CRT : Cathode Ray Tube

# Chapter 1

# Introduction

## 1.1 Uncertainty Visualization

Data sets from most disciplines in science and engineering have ancillary uncertainty information. This uncertainty can refer to many concepts including error, accuracy, variability, noise or completeness of the data and usually arises due to errors in data acquisition and data processing. Since visual analysis is increasingly used to interpret the data, it is important to communicate the associated uncertainty information while visualizing the data. A good visualization method should not only allow the users to explore the data distribution but also attempt to answer queries like

- What are the regions of high or low uncertainty in the domain?
- What is the distribution of uncertainty within a given spatial region of interest?

This assists the users to be conscious of the confidence of interpretations of the data and decisions made based on the visualization.

Uncertainty can be quantified and represented in more than one way depending on its nature [1]. A large class of uncertainty factors such as confidence level, variability or error associated with a scalar value is commonly represented by another scalar. In this project, we focus on 3D scalar fields where every point in the domain is associated with a pair of scalars representing the data value and its uncertainty. Though this representation is simple, we need a systematic visualization method to explore the scalar value distribution with corresponding uncertainty information.

Simple methods such as error bars and box plots are effective for visualizing scalar uncertainty in 1D scalar fields. The problem is challenging for data sets in higher dimensions. Various uncertainty visualization methods have been studied for scalar data over 2D surfaces [2, 3, 4]. But none of these methods are immediately applicable in 3D. Existing methods for uncertainty visualization in 3D often modify a basic visualization method by introducing glyphs or textures [5] to convey uncertainty. These methods can represent only coarse details in uncertainty.

Direct volume rendering [6] is a standard method to visualize 3D scalar fields. It projects the entire volume on to the screen as seen from an interactive viewing position. The user can specify a transfer function that maps each scalar value to a color and opacity and thereby, study in detail the distribution of scalar values in the data set. In this report, we propose a visualization method based on volume rendering that enables users to explore the scalar field along with the associated uncertainty information.

The first step to visualize data is to design a transfer function which is essentially a mapping from the data points to visual attributes like color and opacity. Typically, a color is represented using three primary components namely red, green and blue. Every color has a luminance which is an approximate measure of how bright it appears. The ratio of maximum luminance to the minimum non-zero luminance of colors in an image or a scene is referred to as its dynamic range. There is a limit to the maximum dynamic range of any digital image inherently imposed by the color representation that it uses. If we use conventional color representation with one byte to encode each of the color components, we cannot capture the entire dynamic range of a typical natural scene which is about $10^5 : 1$. HDR imaging uses floating point color components and hence can be used to capture all the perceivable information in most natural scenes. However, a HDR image cannot be directly displayed on a typical display device such as a CRT monitor whose dynamic range is limited. To overcome this limitation, a variety of tone mapping techniques are studied in literature to generate low dynamic range image that fairly depicts a HDR image preserving most of its details.

We use colors in HDR space and design a transfer function to captures fine details in data as well as uncertainty. We then incorporate tone mapping methods in visualization pipeline to enhance details in uncertainty distribution and finally, display the volume rendered image on a conventional 8-bit-per-channel display device.

## 1.2 Organization of this report

The outline of the rest of this report is as follows. In chapter 2, we discuss previous work related to uncertainty visualization and also provide an overview of prior work in HDR imaging. In chapter 3, we explain our uncertainty visualization method in detail and provide implementation details. In chapter 5, we discuss applications of our approach and demonstrate results. In chapter 6, we discuss the contributions of this project and conclude with a discussion on future work.

# Chapter 2

# Related Work

Literature pertinent to our work is found in two different areas of research.

- Uncertainty visualization
- HDR Imaging and HDR volume visualization

We now overview prior work within each category and discuss how it relates to our work.

## 2.1 Uncertainty Visualization

Uncertainty visualization has become an active topic of study within the visualization community ever since its importance was emphasized by Johnson and Sanderson [7].

Pang et. al. [8] demonstrated that a significant amount of uncertainty is introduced in most cases due to usage of different approximation algorithms and choice of interpolation methods. They also summarized a variety of techniques suitable for visualization of such uncertainty. These methods include addition and modification of geometry and attributes, animation, sonification and psychovisual methods. However, these methods are not suitable for a generic 3D scalar data set with uncertainty.

Lodha et al. [9] used glyphs to visualize uncertainty in scalar fields. Wittenbrink et. al. [10] concentrated on vector fields on surfaces and used glyphs for visualization of uncertainty in magnitude as well as direction of a vectors. An important drawback of these methods is that the glyphs can only be placed at discrete grid points and hence do not display detailed variations in uncertainty.

There has been a considerable amount of research on uncertainty visualization of scalar fields over 2D surfaces. The work of Cedilnik and Rheingans [2] describes a technique with minimal interference. They employed procedural techniques to distort geometric primitives like grids that annotate the data. Grigoryan and Rheingans [3] addressed the problem of visualizing surface uncertainty by rendering the surface as a collection of points and displacing each point from its original location along the surface normal by an amount proportional to the uncertainty at that point. Lee and Varshney [4] described the visualization of molecular surfaces whose position is uncertain due to thermal vibrations. They generated fuzzy surfaces by rendering multiple layers of transparent surfaces at different configurations formed by vibrating points. The transparency of a point in a layer was decided by the confidence level of its position.

Though these methods are effective for visualization of surface uncertainty, they are not applicable to volume rendering of 3D scalar fields. Djurcilov et al. [5] identified this drawback and presented a direct volume rendering approach for visualizing scalar volumetric data with uncertainty information. They discussed post-processing of the rendered volume by introducing discontinuities such as speckles, depth shaded holes, adding noise and using textures to represent the uncertainty. Studying detailed variations in uncertainty using this method requires the introduction of lot of noise textures during post-processing which may result in visual clutter. To overcome this, we use colors with floating point components and propose a method for uncertainty visualization based on HDR volume rendering.

## 2.2 HDR imaging and HDR volume visualization

Debevec and Malik [11] introduced the concept of HDR imaging by developing a mechanism to recover and represent HDR radiance maps from a sequence of low dynamic range photographs of a scene captured at different exposures.

Most display devices are not capable of displaying images with dynamic range more than 1000 : 1. Various tone mapping algorithms have been studied in literature to bridge the gap between HDR imaging and low dynamic range display devices. They are inspired by fields as diverse as image processing, photography and human visual system modeling. Apart from reducing dynamic range, the tone mapping operators attempt to provoke same perceptual responses as when viewing a HDR scene in the real world. Tone mapping algorithms can be broadly classified into two categories.

1. **Global tone operators**

   The same transformation is applied to color at every pixel in the image [12, 13, 14, 15]. The transformation is typically non-linear and depends on the properties of the image as a whole.

2. **Local tone operators**

   Dynamic range of the image is reduced by a transformation which is not spatially uniform. These operators exploit the fact that perception of color at a pixel is greatly influenced by the surrounding colors as well. Contrast reduction applied at a pixel is usually determined by its local neighborhood [16, 17, 18, 19].

The benefits of using HDR technology in visualization are yet to be studied extensively. Ghosh et al. [20] used HDR display technology for volume rendering. Yuan et al. [21] used volume rendering with colors in extended dynamic range especially for visualization of high precision scalar volumetric data sets with high spatial resolution. The used floating point color components in transfer function design to allow for high dynamic range colors. They used tone mapping methods to display the volume rendered HDR image on conventional display devices.

In this project, we focus on using HDR technology for uncertainty visualization. We discuss methods to design transfer functions and suitably customize HDR volume visualization pipeline specifically to address the problem of uncertainty visualization.

# Chapter 3

# Our approach

Uncertainty is an important property of the data that determines the confidence of decisions made based on visual analysis. When data is combined with the associated uncertainty, it increases the amount of information that we need to convey visually. Standard scheme for encoding color with 8-bit channels for each of the RGB components forces considerable approximations for both uncertainty and data values, due to which the resulting visualization suffers significant loss in details. We overcome this limitation by using floating point color components employed in HDR technology.

We work with 3D scalar data sets in form of regular 3D grids where a pair of scalars $(\mu_p, \sigma_p)$ is available at every grid point $p$, representing the data value and uncertainty respectively. The scalar and uncertainty at a point elsewhere in the domain is obtained using trilinear interpolation within the voxel containing the point. To visualize such data sets, we devise a method based on direct volume rendering. We also develop an interface where users can interact with the visualization in the uncertainty space. This serves as a powerful tool to make informed decisions under circumstances where the knowledge of uncertainty is crucial. A schematic diagram of our visualization method is shown in figure 3.1. It has two important modules:

1. **HDR Transfer Function design**

   This involves mapping a data point to color and opacity. A good transfer function captures all important details in data and uncertainty.

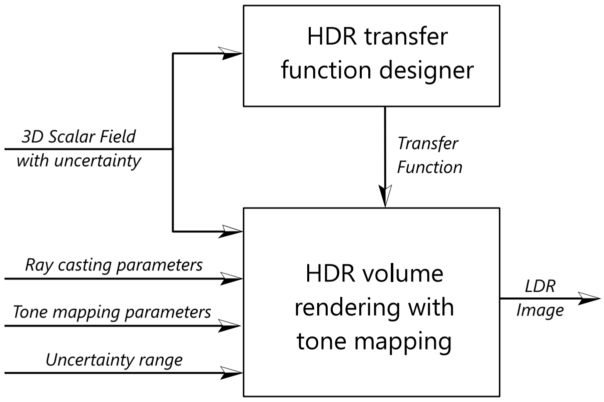



Figure 3.1: Schematic diagram of our visualization method.

2. **HDR Volume Visualization**

   The data set after application of transfer function yields a 3D color volume possibly with a high dynamic range. The HDR volume needs to be rendered on conventional display devices with limited dynamic range. We modify existing tone mapping techniques to work directly in 3D, and thereby reduce the dynamic range of the volume while preserving most details. We use ray casting to project the volume on to a 2D plane as seen from a given viewing position.

We now discuss each of these modules in detail.

## 3.1 HDR transfer function design

The quality of a volume rendered image is mainly determined by the transfer function used. Hence, transfer function design plays an important role in any visualization technique that involves direct volume rendering.

We use CIE Lab color space to encode a color. In this space, a color is represented using a lightness component ($L^*$), a pair of chromaticity components ($a^*, b^*$) and an opacity component ($\alpha$), each of which is a single precision floating point number. The hue of a color depends on its chromaticity components and the luminance depends on its lightness component. Perceptual research indicates that hue plays a major role in

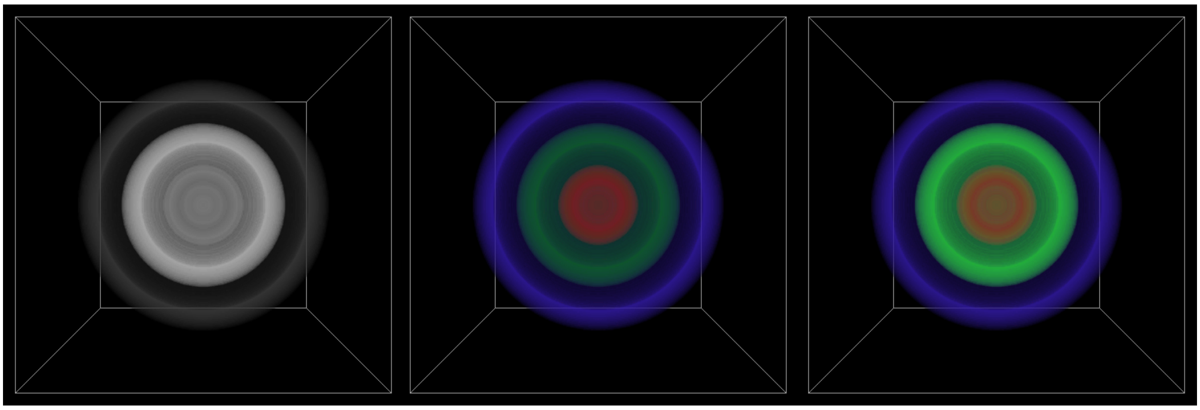



Figure 3.2: Transfer function design

visual grouping. However, if we use different hues to represent data and uncertainty, it results in insufficient number of colors to encode all the information. Therefore, we use hue property to represent only the scalar value. We use luminance component to encode uncertainty. The usage of floating point color components enables us to use wide range of luminance values to encode uncertainty and thus, allows the user to study fine details in uncertainty distribution.

This idea is demonstrated in Figure 3.2. It uses a synthetic data set consisting of three concentric spheres having different scalar values. Different hues distinguish these scalar values. Lightness is defined to be proportional to uncertainty. We can see that the uncertainty of the green sphere is higher compared to the other two.

User can specify mapping from scalar values $\mu_p$ to chromaticity $(a_p, b_p)$ and opacity $\alpha_p$ components. Depending on the kind of data being visualized, suitable methods can be used to define this mapping. For example, if we are interested in identifying the structure of different materials in the volume, we can use a multidimensional transfer function to enhance material boundaries [22]. In this case, the chromaticity and opacity of the color depends on the scalar value, possibly its gradient, and other parameters based on the scalar value distribution.

All colors that can be rendered have a lightness value in the range [0, 100]. However we allow arbitrarily high values for lightness. We determine lightness component $L^*_p$ of the color based on uncertainty $\sigma_p$ at point $p$. For generic data sets, we observe that it is useful to define lightness as follows.

$$L_p^* = 50 + C * \sigma_p$$

The constant $C$ is interactively set by the user and determines the visual effect of the uncertainty field. When $C$ is zero, all points are rendered with a constant lightness 50. Most hues are well defined for this value of lightness, and resulting visualization is a simple volume rendering of the scalar data for user specified transfer function. As the value of $C$ increases, uncertain regions are mapped to brighter colors.

To help better understanding of the data, we allow the users to specify a uncertainty range of interest. The points that do not have uncertainty in this range are made invisible by mapping to a transparent color ($\alpha_p = 0$).

Depending on the application domain and kind of queries posed on the uncertainty distribution, there may be better ways to define the mapping for lightness, in which case, the user can be allowed to customize this mapping.

## 3.2 HDR volume rendering

We use ray casting [6] to project the volume onto the view plane. Since we are using colors in HDR space, it is necessary to include tone mapping in the visualization process to produce low dynamic range images without any significant loss in details.

A simple way to achieve this is to use the HDR volume visualization pipeline [21] introduced by Yuan et al. It involves casting a ray through the volume for every pixel in the view plane, and blending colors along each ray to obtain the color at the pixel. The dynamic range of the resulting image is determined by the distribution of uncertainty in the volume, and hence can be very high. We can then use existing tone mapping algorithms to reduce this dynamic range and display the resulting image on screen. This model is shown in Figure 3.3.

However, the design of our transfer function enables us to make some improvements

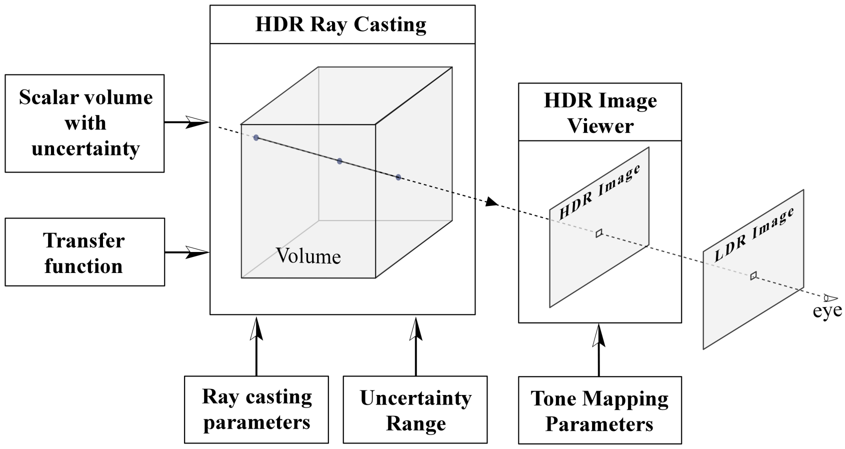



Figure 3.3: HDR Ray Casting

to this model when used for uncertainty visualization.

Since $L_p^*$ is defined as a linear function of $\sigma_p$, it can be obtained by trilinear interpolation of lightness values associated with corners of the voxel containing $p$. This is not true in general for simple ray casting with user given transfer function. However in our case, we can initially obtain a HDR lightness field by mapping every grid point into a lightness value. We can then apply tone mapping on this 3D field instead on applying it on final image. The result is a regular 3D grid of lightness values which constitutes a low dynamic range lightness field with most of its details preserved. While ray casting, we obtain $L_p^*$ value at $p$ using this tone mapped lightness field. This model is shown in 3.4 and helps us improve upon the earlier model in following respects.

- Typically, a tone mapping algorithm applies a parametric function on color at every pixel in the 2D image to obtain a low dynamic range image which can be displayed on screen. Most of the existing algorithms adaptively configure themselves and choose optimal parameters adaptively depending on the input image to preserve most details in the resulting low dynamic range image. Though we have a 2D image as a result of ray casting, it is essentially the visualization of the volumetric data. If the user changes viewing position or interactively transforms the volume, the volume renderer produces a different 2D image. If we use different set of tone mapping parameters for these two images, the resulting images may not be

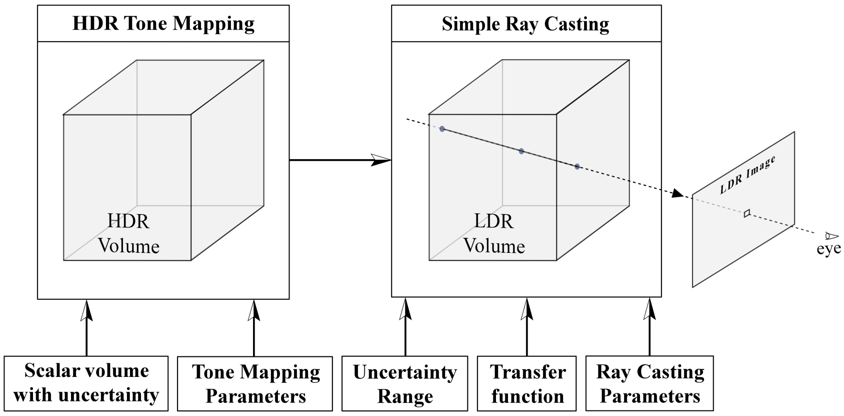



Figure 3.4: HDR Ray Casting for uncertainty visualization

consistent. So, it is important to use tone mapping parameters that are based on the properties of the volume.

- Local tone operators rely on the properties of neighborhood of a pixel and apply suitable transformation on the pixel. When the points are in 3D domain, it is better to study the neighborhood properties in 3D.

- In our model, tone mapping is applied only when the transfer function changes. Since, we do not have to apply tone mapping every time the user interacts with the volume, it allows for faster interaction with the volume. In fact, the ***pre-shading*** of lightness values followed by tone mapping, together with user specified transfer function for hues can be thought of as a simple transfer function.

Our tone mapping method for 3D is based on the algorithm due to Durand and Dorsey [17]. We perform contrast reduction on log of $L^*$ values because the differences in logarithmic scale correspond directly to contrast ratios.

$$L_p = \log L_p^*$$

Edge preserving bilateral filter is applied on the $L$ values to obtain a *base field*, $\tilde{L}$.

$$\tilde{L}_p = \frac{1}{k(p)} \sum_{q \in \Omega} f(||q - p||) g(L_q - L_p) L_q$$

where $\Omega$ is the 3D domain and $k(p)$ is the normalization factor,

$$k(p) = \sum_{q \in \Omega} f(||q-p||)g(L_q - L_p)$$

Bilateral filter effectively blurs the input while preserving sharp edges (or surfaces). We use Gaussian functions for $f$ and $g$ in spatial domain and lightness domain respectively. Further, we restrict $\Omega$ to be the set of grid points in the local neighborhood of $p$ as the other points do not contribute significantly to the summation.

The difference between $L_p$ and $\tilde{L}_p$ is referred to as detail at point $p$. The detail field contains most of the fine details in the distribution and its dynamic range is typically low. We apply dynamic range reduction only on base $\tilde{L}$ values. The detail field is added back to tone mapped base field to obtain final low dynamic range lightness values. As a result we have a low dynamic range volume formed by lightness values defined at every grid point. Trilinear interpolation is used to obtain lightness at an arbitrary point in domain. We now perform a simple ray casting on the scalar field, by obtaining chromaticity at a point from the user defined transfer function. The resulting image is a LDR image and is directly shown on screen.

## 3.3 Implementation details

We have developed a simple application to demonstrate the usability of our method. The scalar field and corresponding uncertainty field are loaded in the form of rectilinear 3D grids from separate files having the same dimensions. Histogram of the scalar values is provided to the user. We obtain mappings for chromaticity and opacity from the user as a function of scalar values. Figures 3.5 and 3.6 show the user interface provided for color selection and transfer function design respectively.

Lightness values are initialized at grid points and tone mapping is applied and the result is served as input to the ray casting module. Ray casting is implemented in hardware using OpenGL fragment shaders. Our volume renderer consists of two phases.

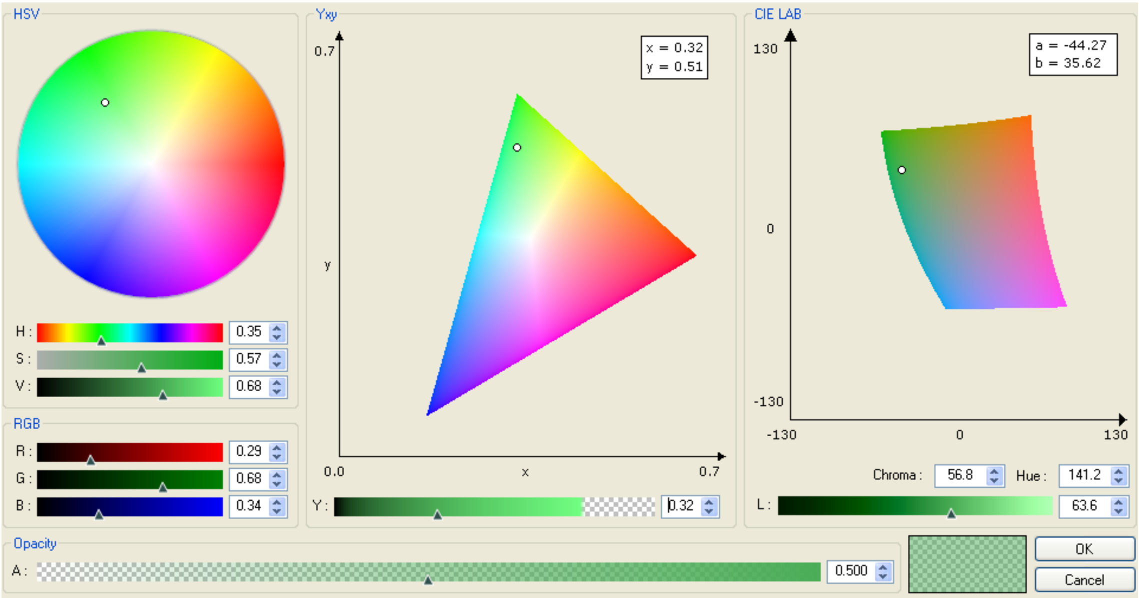



Figure 3.5: User interface for color selection

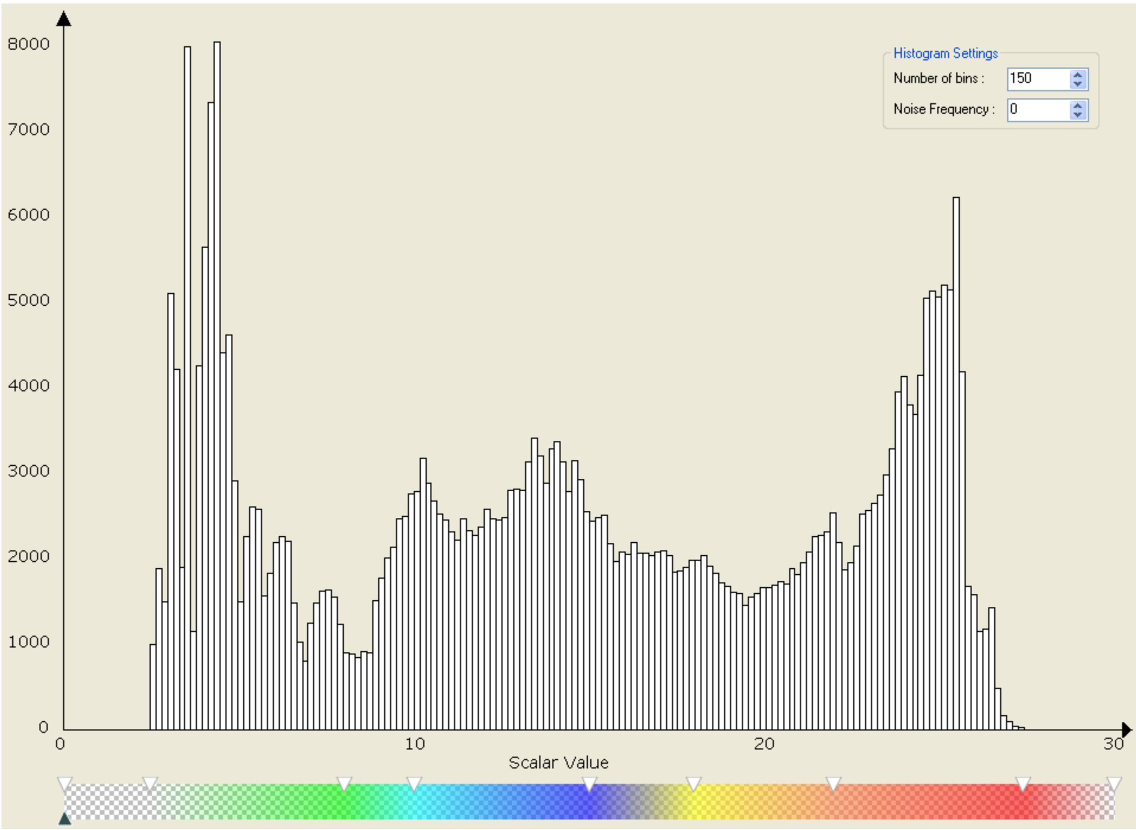



Figure 3.6: Transfer Function Designer

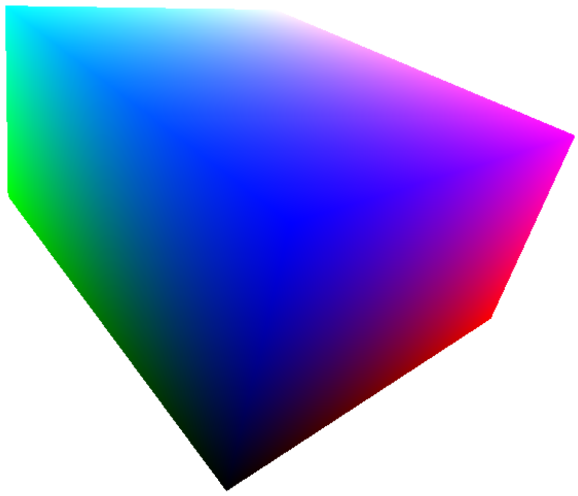


Figure 3.7: Texture coordinates of the starting and ending points of rays

1. **Identification of points of intersection of ray with the volume**

   Corners of the bounding box of the volume are assigned colors with RGB components same as coordinates of the corners of a unit cube. The faces of the bounding box are rendered with smooth shading and back face culling enabled. The color at a pixel represents the texture coordinate of the starting point of the ray through the pixel. The faces are rendered again with front face culling to obtain ending point of rays. The resulting images are stored as textures and are provided as input to the next phase. We use floating point color components to get the accurate position of the points. Figure 3.7 shows typical images with starting and ending points of rays encoded as colors.

2. **Ray casting**

   We use OpenGL fragment shader to implement ray casting. The scalar field, uncertainty field, starting points of the rays and ending points are stored as textures in the graphics memory. For every pixel in the view plane, we sample points at regular intervals on the line segment between the starting and ending points of corresponding ray. For each sampled point $p$, if its uncertainty $\sigma_p$ is not within user specified threshold we map to a transparent color by setting $\alpha_p = 0$. Otherwise, we obtain its chromaticity $(a^*_p, b^*_b)$ and opacity $\alpha_p$ from the transfer function. If the user has requested to use an achromatic color, we do so by assigning $(a^*_p, b^*_b)$ to that of reference white. Lightness $L^*_p$ of the color is obtained using trilinear interpolation on tone mapped lightness values at grid points. The colors along each ray are blended using volume rendering integral [6] to obtain the final color at the pixel.

Color blending is performed in RGB space as it is an additive space and is best suited for scaling and composition of colors.

Users are allowed to interact with the visualization in various ways. The effect of uncertainty on visualization can be controlled by changing the proportionality constant used in transfer function. The volume can be explored in uncertainty space by specifying the uncertainty range of interest. Users can turn off the effect of scalar values to study the distribution of uncertainty within the specified range by disabling hues.

# Chapter 4

# Results and discussion

We use ocean field data set [23] to demonstrate our visualization method and evaluate the results. The data consists of physical variables including temperature and salinity measured on the Middle Atlantic Bight (MAB) shelf break, off the east coast of the United States. Measurements are available at one hour intervals and at 27 different depth levels. Data within each level is sampled on a regular 150x175 sized grid. We consider hourly samples over a day and consider the mean and standard deviation of the measurements as scalar field and its uncertainty respectively. In this context, uncertainty refers variability of the scalar values over time.

While ray casting, we use step size of about 10% of voxel dimensions to generate high quality images with resolution 600x600. We are able to achieve frame rates of about 15 frames per second on NVidea GeForce 8400 GS graphics card.

Figures 4.2 and 4.3 show the results on ocean salinity data and ocean temperature data respectively. The images in the left most column present views of the data distribution without considering uncertainty. They are rendered with every data point mapped to a constant lightness. We can see that the hue channel can effectively capture details in the scalar value distribution. The images in the middle column are rendered using an achromatic color. The patterns in lightness convey the distribution of uncertainty in the volume. The rightmost column shows the visualization of the scalar field with uncertainty and can be used to study detailed variations in data as well as uncertainty.

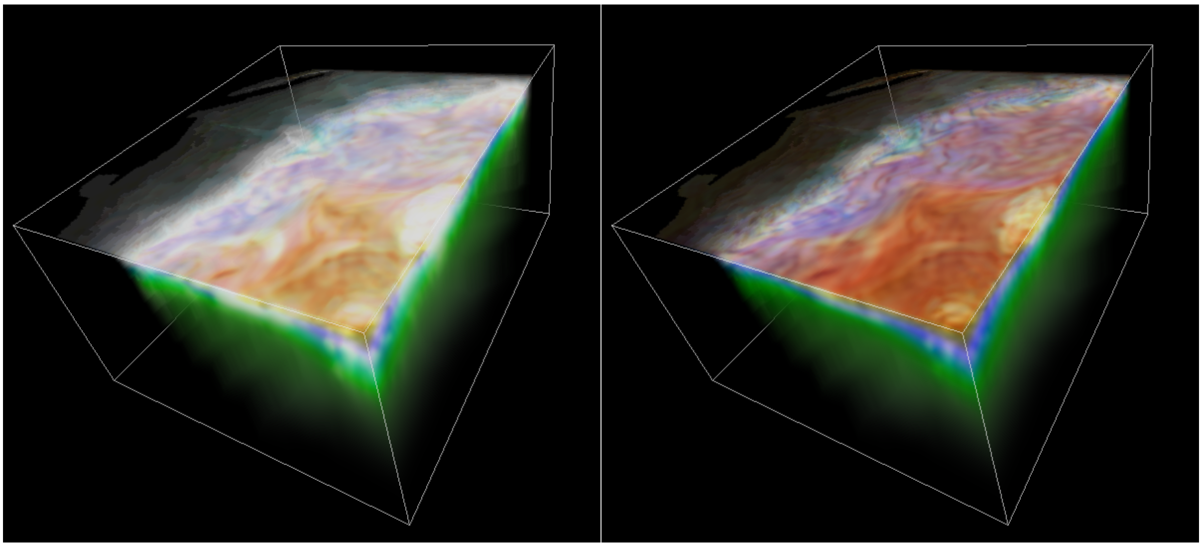
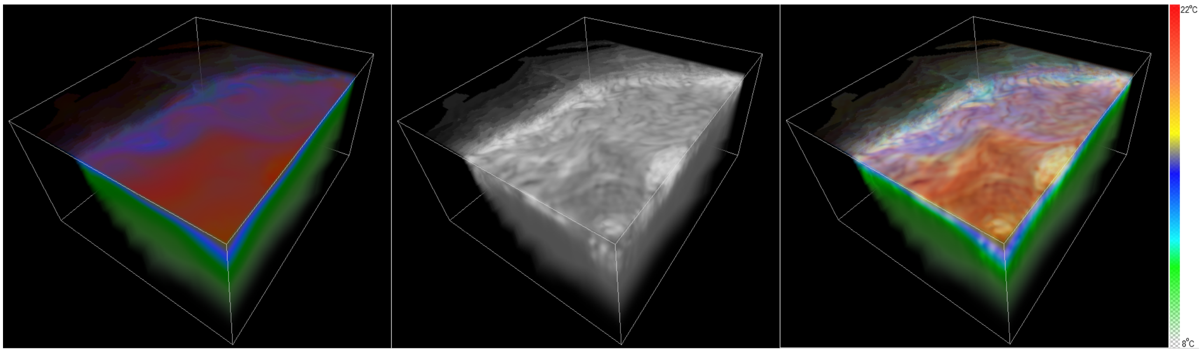


Figure 4.1: (Left) Visualization of ocean temperature field without tone mapping, (Right) Visualization of the same volume with tone mapping.

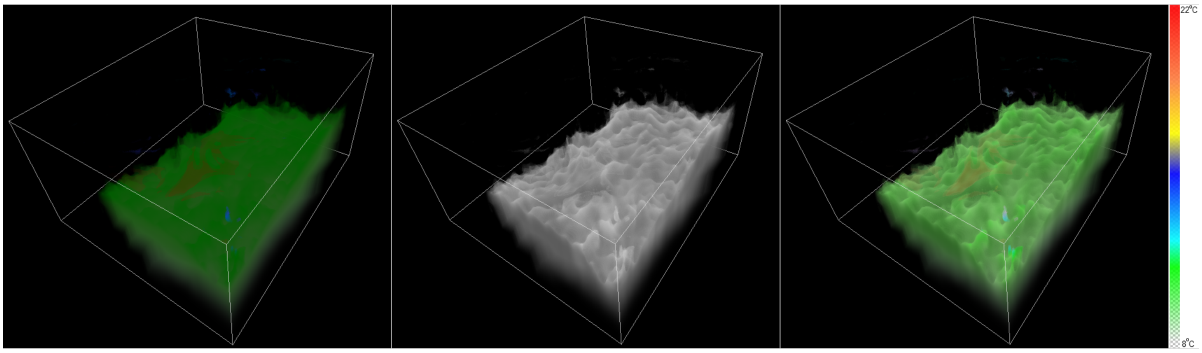
From the visualization, we can infer that temperature uncertainty on the surface is higher compared to deeper levels. In case of the salinity field one observes that the uncertainty is high at the MAB shelf drop-off. Further, as the user interactively increases the proportionality constant used in transfer function, the uncertain regions tend to saturate. This control enables users to explore the data distribution with uncertainty. Users can perceive depth information by rotating and scaling with the volume using the virtual trackball interface.

User can choose to use simple scaling of luminance values instead of tone mapping. Figure 4.1 shows the ocean temperature field rendered with simple scaling and tone mapping based on bilateral filtering.

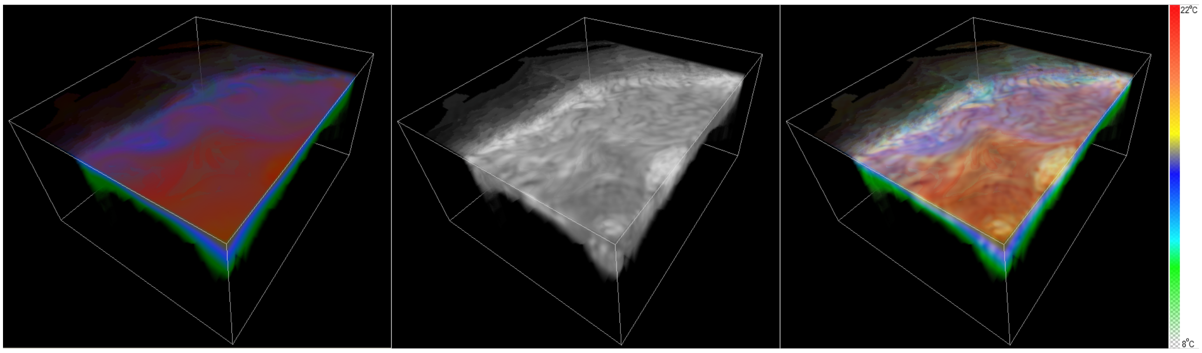
To enable explorations in uncertainty space, we allow the users to specify a range of interest for uncertainty. This helps users to identify the regions of high (or low) uncertainty. Second and third rows in figures 4.2 and 4.3 show the regions of low uncertainty and high uncertainty respectively. The patterns in lightness correspond to variations in uncertainty within the specified region of interest.



(a) Ocean temperature field

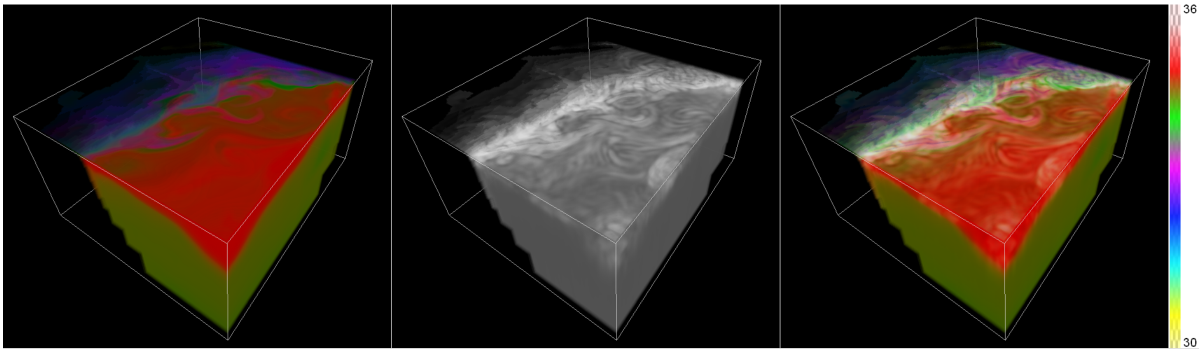


(b) Low uncertainty regions (0.0°C to 0.04°C) in ocean temperature field

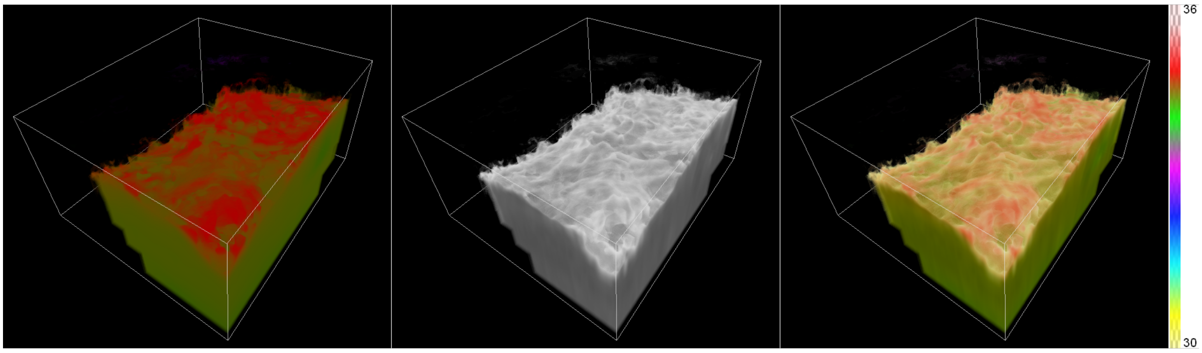


(c) High uncertainty regions (0.04°C to 3.53°C) in ocean temperature field

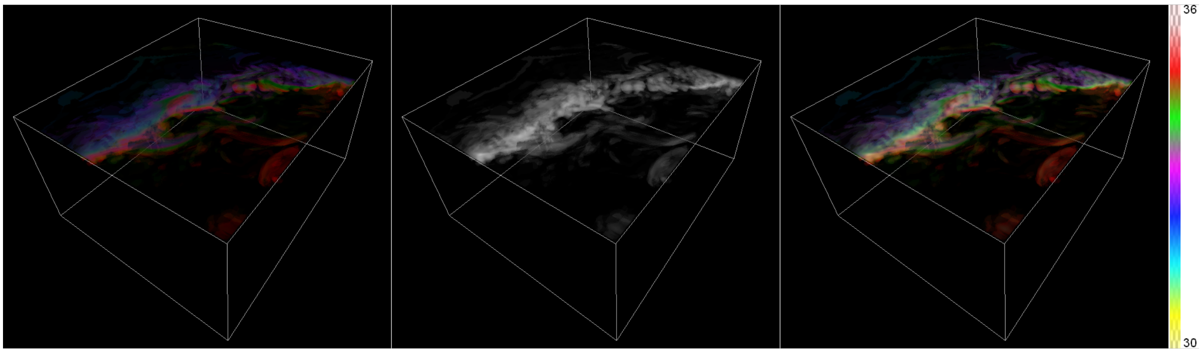
Figure 4.2: (Left) Temperature values are encoded in hue channel of the colors, (Middle) Uncertainty distribution is represented by lightness, (Right) Visualization of ocean temperature field with uncertainty




(a) Ocean Salinity field




(b) Low uncertainty regions (0.0 to 0.01) in ocean salinity field




(c) High uncertainty regions (0.05 to 0.7) in ocean salinity field

Figure 4.3: (Left) Salinity values are encoded in hue channel of the colors, (Middle) Uncertainty distribution is represented by lightness, (Right) Visualization of ocean salinity field with uncertainty

# Chapter 5

# Conclusions and future work

We have developed a novel approach to visualize uncertainty in scalar volumetric fields that allows users to explore the data in both uncertainty space and the scalar data space. The key contributions of this project are listed below.

- We explored a novel idea of using HDR technology in uncertainty visualization to encode data as well as uncertainty in floating point color components.
- In our transfer function, we used separate mappings for lightness and hue channels of color. This design enabled us to improve upon existing HDR volume visualization pipeline by using pre-shading and tone mapping of lightness values in 3D.
- Our method is purely based on volume rendering is able to display detailed variations in data as well as uncertainty without using uncertainty glyphs or noise textures. Our method is also applicable to slices, isosurfaces, and for scalar fields in lower dimensions.

We have demonstrated the applicability of our approach using data sets from ocean modeling. As future work, we would like to explore alternate transfer function designs tailored to specific applications. We would like to apply our method to analyze uncertainty characteristics in the context of specific measurement devices. We believe that using HDR technology to enable perception of details in visualization can be extended to other attributes besides uncertainty.

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