Eurographics Conference on Visualization (EuroVis) 2026
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(Guest Editors)

COMPUTER GRAPHICS *forum*
Volume 45 (2026), Number 3

Uncertainty-Aware Visual Analysis of Force Networks in 2D Granular Materials (Supplemental Information)

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1. Alignment of Measurements

The mapping between the disks of different realizations is obtained by assigning a unique ID to each disk. As the initial condition is repeated for each measurement, the results are close enough such that the positions of the nodes can be directly used to assign the same IDs for each of the experiments. Thus, we choose the first realization as a reference to which we assigned the individual IDs and assigned IDs for realization i as

$$v_{i,j}.id = v_{1,k}.id \text{ with } k = \arg\min_{l \in |V|}(|v_{i,j}.p - v_{1,l}.p|), \quad (1)$$

where $v_{i,j}.id$ denotes the ID of node j in realization i and $v_{i,j}.p$ is the position of node $v_{i,j}$. For mapping over the changes of the packing fraction, we note that differences between disk positions in realizations of consecutive steps are very small. We use the same mapping as in Equation 1, but map to the previous packing fraction step of the same measurement instead of the first step of the first measurement. We observe only a single case where the automatic mapping does not lead to correct results and which we correct manually. Alternatively, more advanced matching methods such as the Hungarian algorithm for solving the assignment problem [Mun57] could avoid manual correction but come at a higher computational cost.

2. Additional Information on the Domain Expert Evaluation

In the following, we provide more details on how we conducted the interviews with the domain experts.

2.1. Formative Evaluation

In a first online meeting, we started with a semi-structured interview consisting of questions regarding their previous analysis, their current treatment of uncertainty, and their analysis tasks. After that, we provided a brief introduction (ca. 20min) where we explained our visualizations based on a subset of the entire dataset. The domain experts were invited to ask questions to clarify any misunderstandings. In the next step, we conducted a pair analytics session, where an initial analysis of the data was performed using the full dataset. After that, we conducted a semi-structured interview, where we asked questions about the individual visual encodings and collected feedback on what else they would need, how it would fit into their analysis workflow, and if they had any additional comments. The total meeting took 1 hour.

2.2. Summative assessment

The input obtained in the first interview was used to enhance our visual analytics solution, especially with domain-specific terms and annotations, such as marking the jammed region. The final approach was deployed as a web application to provide full access to domain experts and was accompanied by documentation materials that show the usage and interaction capabilities. Over the entire process, additional questions were clarified using asynchronous communication. In two further meetings (one hour each), we performed a semi-structured interview to obtain insights into their findings about the data and obtain a better understanding of where the visualization approach is useful, as well as its weaknesses. In the first meeting, the domain experts proposed the aggregated spatial view, which we included between the meetings.

3. Computation Times

For estimating the computation times for different data sizes, we parametrize the dataset described in Section 6.1 such that the number of disks, the number of steps, and the number of samples can be adapted. By default, we use 625 disks, 5 steps, and 10 samples, and vary each parameter individually. The results are shown in Figures 2,3, and 4. For an interactive analysis, analytical rates are important to avoid long waiting times. For the largest datasets with 40 steps, 625 disks, and 10 samples, computations take less than 10 seconds. For the real-world dataset, the computations of the average node degree, the number of non-participating nodes, and the average connected component size take on average 8.4 ms, 4.3 ms, and 271.3 ms, respectively. However, these measures need to be computed several times, which is why the heatmaps are precomputed. The spatial view can be computed in 1.9 s, and the Sankey computation takes 7.1 s during initial data loading. Recomputations during interactions are performed on demand, reducing computation time.

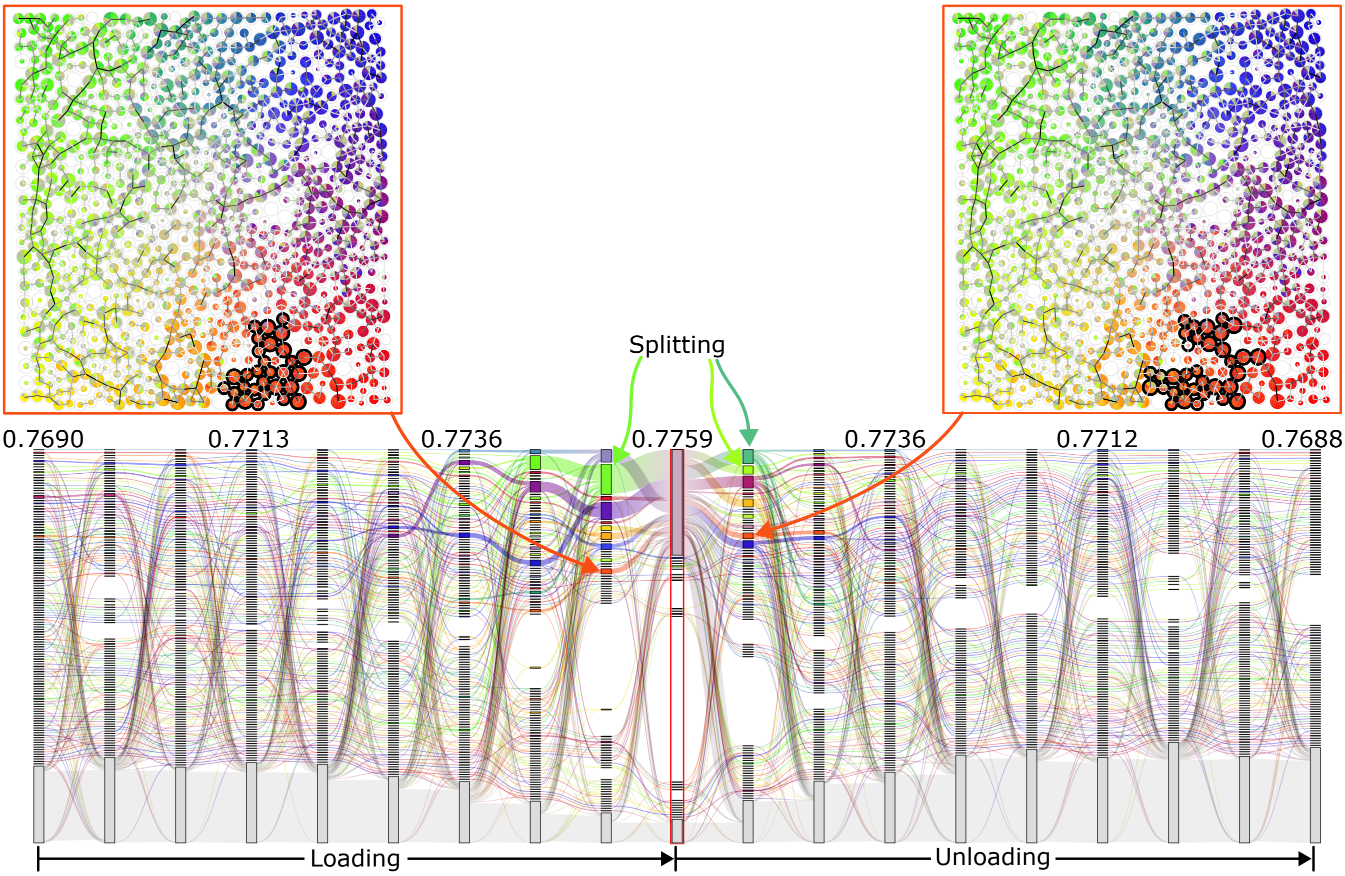

Figure 1: The Sankey diagram filtered by $F_t = 0.21$ and $P_t = 28\%$ shows smaller structure. The components are not perfectly symmetric during loading and unloading. The green arrows indicate an example where the components differ. Also, structurally similar components appear (see insets).

For example, the spatial view can be computed in less than a second when changing the selected step.

While there is a strong variation, computation times are mostly asymptotically constant or grow linearly with the corresponding parameter. The only exception is the number of disks, which leads to a faster increase in computational complexity. This can be explained by the quadratic number of edges in a graph.

4. Visualization Expert Study

Figures 5–9 present the detailed results of our study that we conducted with 16 visualization experts. Please refer to Section 6.4 of the main paper for details.

References

[Mun57] MUNKRES J.: Algorithms for the assignment and transportation problems. *Journal of the Society for Industrial and Applied Mathematics 5*, 1 (1957), 32–38. 1

[WAM*19] WALL E., AGNIHOTRI M., MATZEN L., DIVIS K., HAASS M., ENDERT A., STASKO J.: A heuristic approach to value-driven evaluation of visualizations. *IEEE Transactions on Visualization and Computer Graphics 25*, 1 (2019), 491–500. doi:10.1109/TVCG.2018.2865146. 4

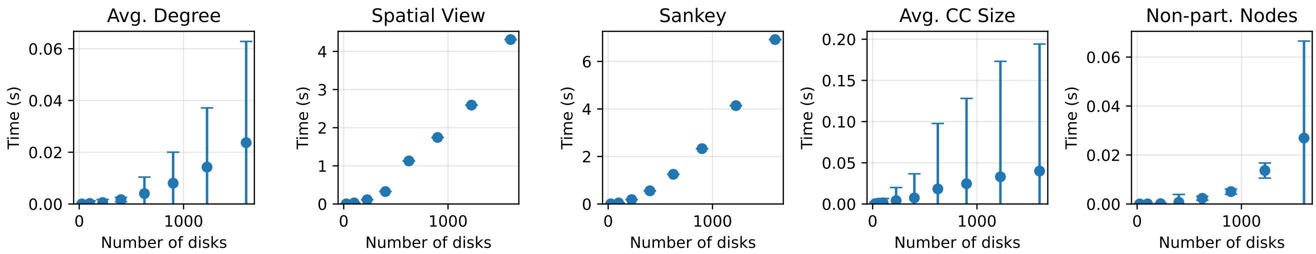

Figure 2: Computation times for the parametric dataset with changes in the number of disks. Error bars indicate the variation among the measures that are computed several times.

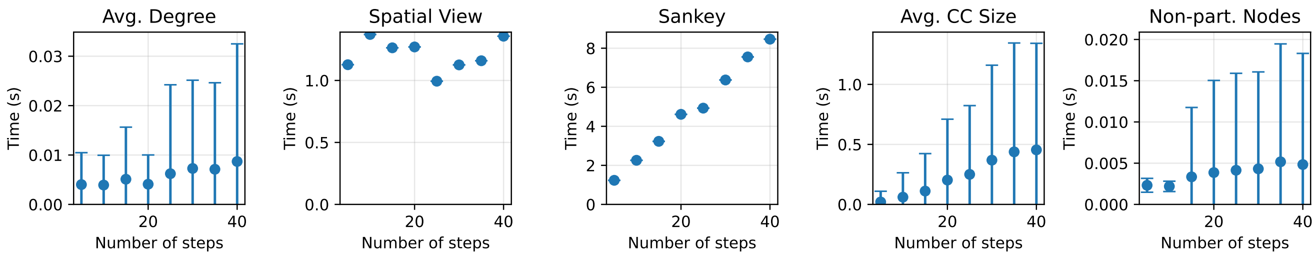

Figure 3: Computation times for the parametric dataset with changes in the number of steps for different packing fractions. Error bars indicate the variation among the measures that are computed several times.

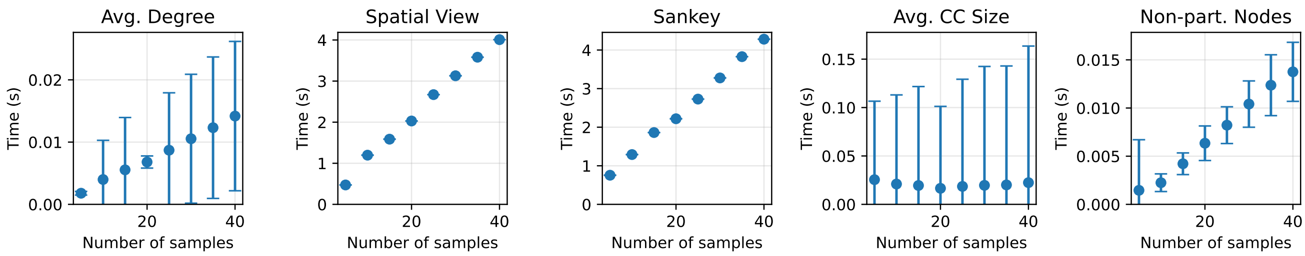

Figure 4: Computation times for the parametric dataset with changes in the number of samples. Error bars indicate the variation among the measures that are computed several times.

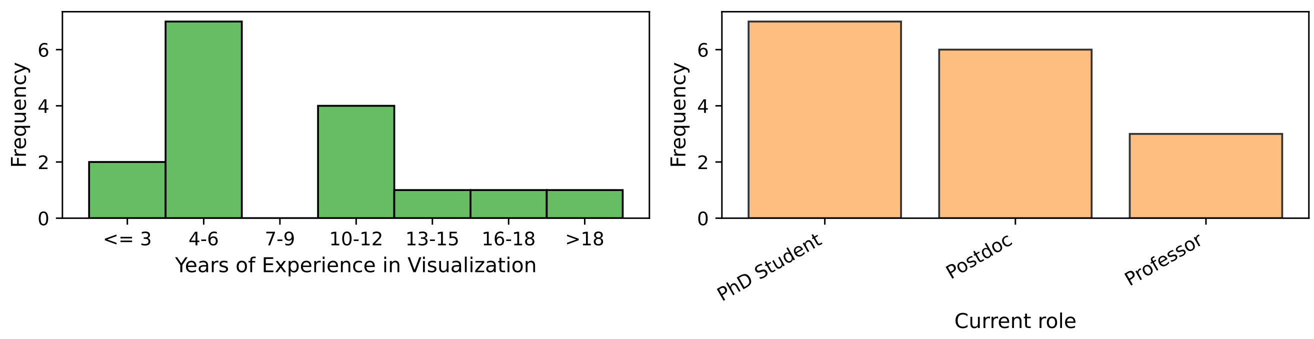

Figure 5: All of our participants had at least 2 years of experience in visualization, but most have worked in the field for many years, such that it is likely that they have seen different visual analysis approaches.

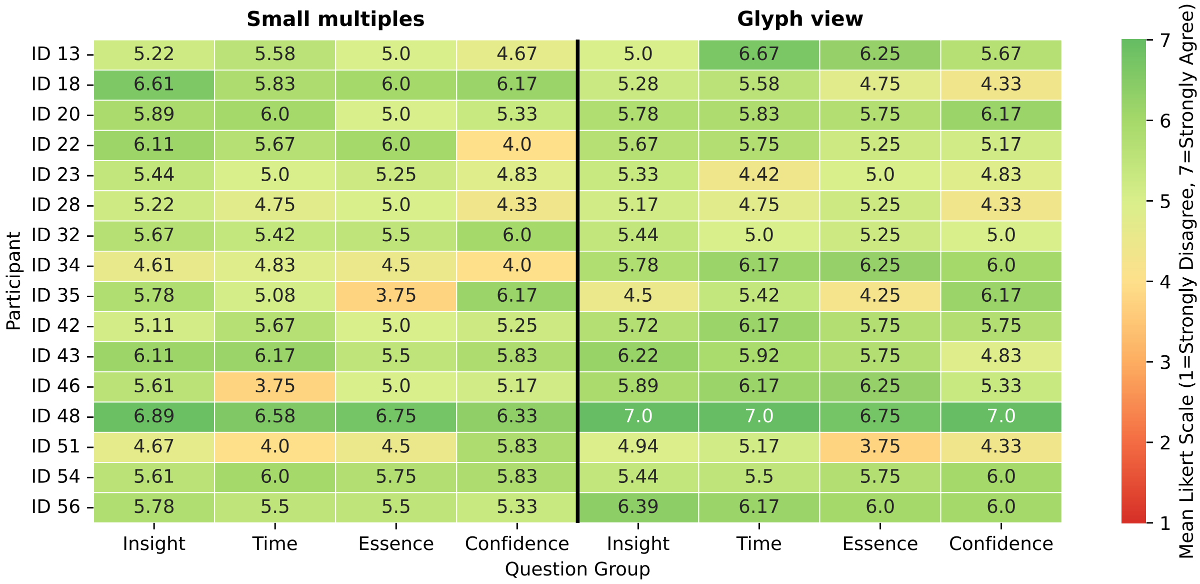

Figure 6: The scores for the different categories are computed as described by Wall et al. [WAM*19]. Both visual encodings show overall positive scores.

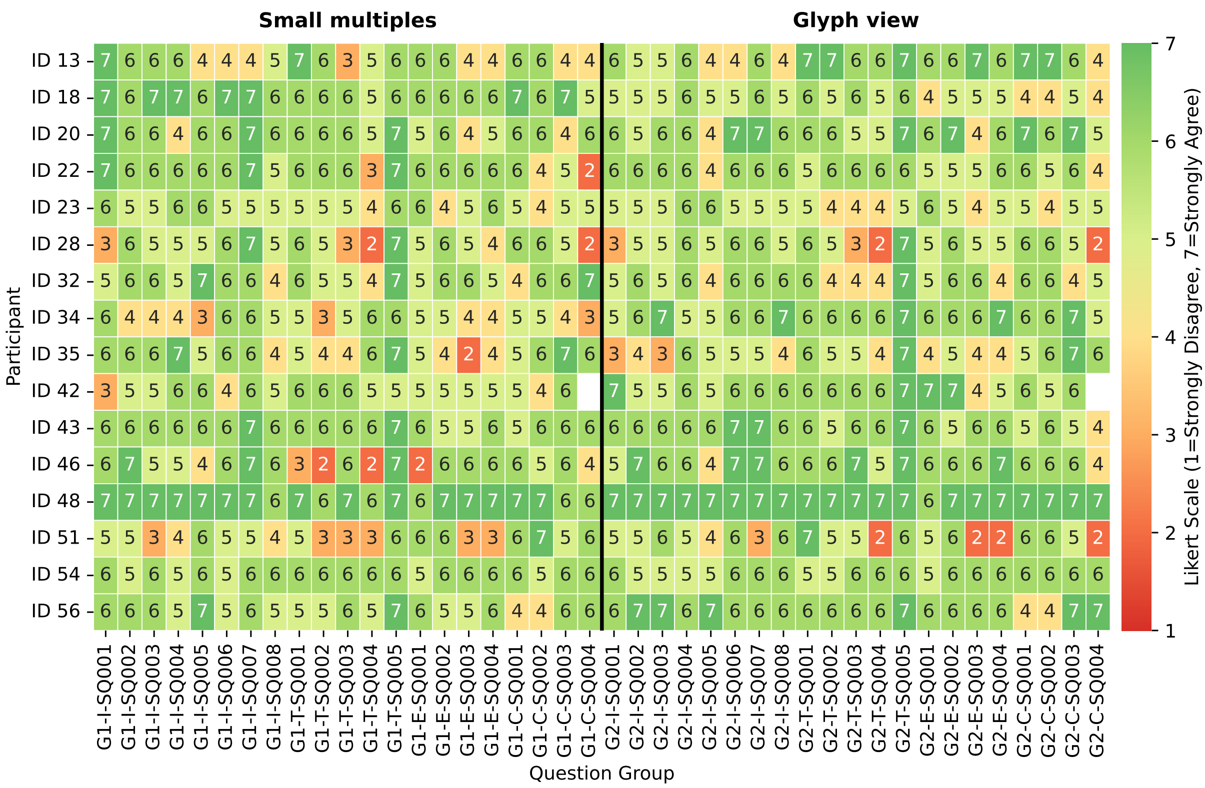

Figure 7: The heatmap shows the individual responses of the 16 participants for both visual encodings.

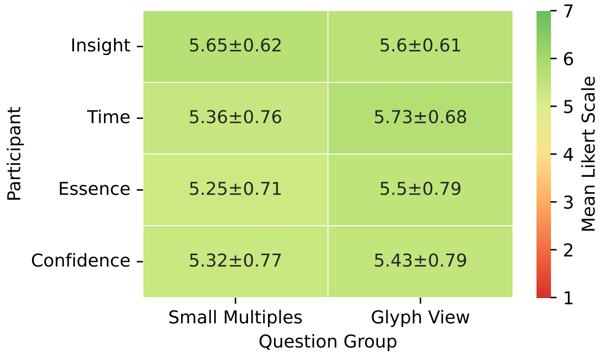

Figure 8: The individual values of the Likert Scale (1 corresponds to strongly disagree to 7 to strongly agree) are aggregated for the different groups. All values are similar, indicating that the participants rate these aspects positively. The annotations indicate the mean and the standard deviation.

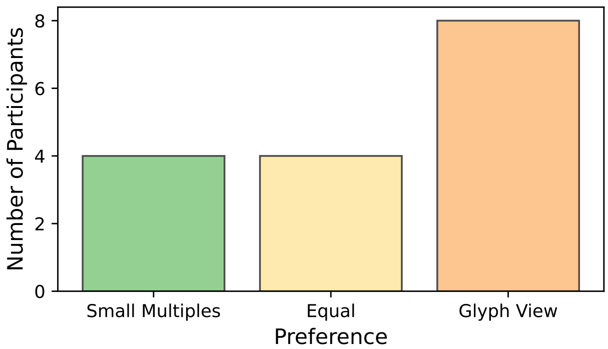

Figure 9: After filling out the ICE-T questionnaire, the participants were asked which view they considered more suitable based on the visualization tasks presented in the paper.

<u>Domain Expert Interview (1. Meeting)</u>

Setup:

- Ask for permission to record the interview
- Introductory questions
- Introduction to approach
- Pair-analytics session: I share my screen, discuss, have them ask questions
- Remaining questions

<u>Questions:</u>

Before introduction:

General questions:

- Did you already do a joint analysis of all datasets? If yes, how?
- When analyzing multiple measurements of granular material data, how do you refer to or describe the uncertainty or variability in the results? (e.g., "noise," "error," "spread")
- Is there something specific that you aim to look for when trying to understand the variability in the data? (e.g., patterns, outliers, trends)

After the introduction and pair-analytics session:

Questions on individual visualizations:

- Would the glyph-based visualization be helpful?
- What would you like to understand for a visualization for a single packing fraction?
- Are the following measures of interest for you:
    - Number of non-participating nodes
    - Average node degree
    - Size of connected components
- Are there alternative measures that would be relevant?

Questions about our results:

- Are there any specific features or aspects of the visualizations that you find particularly useful or confusing?
- Would you prefer starting on the more detailed level (glyph visualization) or with an overview over different packing fractions?
- Would you be interested in visualizing cycles (like in Farhans work) in a similar manner?

Closing:

- Is there any further information that you would like to include in the analysis?
- What would it need to be a usable system for you? Would you like to use it as it is?
- Where would the visual analysis system fit in your workflow?
- Is there anything that we haven't discussed yet?

Domain Expert Interview (2. Meeting)

Record the interview again: ask for permission

Questions about their usage of the approach:

- What was your impression of using the tool? (Goal: get some general impressions)
- Is there something that you found very easy or very difficult to understand?
- Are there visualizations that you relied most on and why?
- How do the different visualizations align with how you would analyze you data otherwise?
- Could you imagine using a similar approach for your regular data analysis?

Use Case

- Ask for findings/something surprising which they could show to me, and which would be helpful as an example in the paper

Ask questions that come up based on what they say

Closing:

- Ask if there is something from their side to discuss
- Discuss paper plans and next steps
  - Ask them to check the domain-specific information and if that is all correct
  - Tell them that I will contact them after writing the evaluation and ask them to have a look at it if that is ok

-

Questionaire

- NASA-TLX
- Other questions targeting the different perspectives (answer on Likert scale):
  - In the line charts, the uncertainty is shown as variance. How intuitive does that feel for you?
  - In the heatmaps, the uncertainty is treated as a variable for the probability which can be used for thresholding in the Sankey diagram. How intuitive does that feel for you?
  - In the spatial view, uncertainty is used to show the likelihood of force chains and the probability of edges between disks. How intuitive does that feel for you?
  - The different views are combined in a single tool. How intuitive is the combination of the different visualizations?

Study description

In the beginning, you will receive an introduction to the data and to the visual analytics approach. For the main part of the study, you will receive links to the visual analytics tool which you can test and explore. After exploring the tool, you will assess the visualizations by answering a questionnaire with 21 questions. This procedure is repeated with a different visualization. In the end, you will answer 4 questions regarding your previous experience.

A full evaluation for the intended purpose is only possible when the entire questionnaire is complete. However, you can cancel and leave the study at any time. In this case, we will delete the already collected data.

Please participate in this study on a computer and not on a tablet or phone!

For any questions regarding this study, please contact Marina Evers (marina.evers@uni-siegen.de).

Section A: Introduction

Data and Domain Description

Granular materials, such as sand, form a complex system in which the forces between individual particles influence the material's macroscopic behavior. The data shown in the following visualizations originates from physical experiments on 2D disks. During the experiment, the particles are gradually compressed and decompressed (changing the so-called packing fraction), causing forces to form between neighboring particles.

These forces create a force network, where particles are connected if they exert a force on each other. The same experiment is repeated multiple times under similar conditions. As a result, the presence and strength of forces, as well as larger structures formed by them, can vary between repetitions.

The dataset therefore captures not a single outcome, but variability across repeated measurements, allowing the analysis of both common structures and their uncertainty.

The main tasks for the visualizations are:

T1: Identifying reliable structures: The goal is to find information with a high probability of occurring in a measurement. Structures that only rarely occur should be visualized while also communicating the uncertainty, such that the probability is conveyed.

T2: Visualizing different scales: The main goal of the experiment is to obtain an understanding of how structures on the microscale, such as forces between particles, relate to information on the macroscale, such as the number of nonparticipating nodes in the force network.

T3: Investigate the behavior when applying pressure: With increasing pressure, the packing fraction of the system increases. To better understand phase transitions in granular materials, the behavior of the system and associated uncertainty should be visualized as the packing fraction changes.

Visual Analytics Tool

For the following questions, you will receive links to a visual analytics tool with two different visual encodings to the spatial structure (small multiples and a glyph-based visualization). Both approaches work very similar and support the same interactions. You will be able to access this information again later for reference.

You can watch an overview video on the tool: https://uni-siegen.sciebo.de/s/nqKz3E7w9N6WTPL

In the following, you find a detailed description of the views and the interactions:

Spatial view:

The spatial view varies between the two different tools you will encounter in this study. In both cases, it is designed to find force chains which are line-like structures in the networks and can be described by connected components of the force network. As illustrated in the following figure, we extract them for each measurement individually and assign a color based on the barycenter of the connected component. The resulting components are shown individually as small multiples or combined in a glyph visualization.

Interactions:

- Clicking on a glyph/a disk highlights all disks that are connected to the selected disk. In case of small multiples, all disks that belong to the connected component are highlighted.

- Clicking outside of a disk removes the selection.

- Show Edges - Toggle: shows or hides the edges of the force network. In the aggregated view, a darker edge indicates that this edge occurs in a higher percentage of measurements.

- Order by color similarity - toggle: If on, the slices of the glyphs are ordered such that similar colors appear next to each other, otherwise the original ordering of the measurements is used.

- Use size for probability - toggle: If off, light grey slices are inserted if the disk is not part of a connected component in the force network. Otherwise, the area of the glyph is shrinked to encode in how many measurements the disks participates in force structures.

- Show major/minor - toggle: Allows to show domain specific information, major force chains (red) and minor force chains (blue)

- Numbered buttons (left): For the glyph view, the allow to show the individual measurements

- Button D: Show the deviation in disk positions

Linecharts and Heatmaps:

The label in the left indicates what is shown in both, the line chart and the heatmap. In total, three different measures are shown:

- Avg. coord. number: The average coordination number corresponds to the average degree of a node in the network (on how many neighbors does a disk exert a force)

- Mean CC size: The average size of a connected component

- Non-participating nodes: Number of disks that are not experiencing any force by any other disk

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Section B: Small Multiples

The initial loading may take some time. The application is loading as long as you browser tab states "Updating...". You can freely explore the visualization. Then, please answer the following questions based on the visual analysis approach.

Note that the toggles beside the small multiples view do not have an effect.

Please access the visualization here: https://uncertainty-cycles.cg.informatik.uni-siegen.de/dash/?view=small (if you do not see several small multiples in the top-left, please try Firefox or Safari as a Browser)

If you would like to access the introduction again, you find the information here: https://uni-siegen.sciebo.de/s/tMKnoqqiTnqAY3S

Terminology:

data case- refers to an instance of the data set; synonymous with data item or data point

attribute- refers to properties of the data cases in the data set; synonymous with feature, dimension, or variable

relationship in the data- refers to attributes among the data, such as correlations, clusters, or distributions

B1. Insight

How would you rate your agreement with the following statements?

	Strongly Disagree	Disagree	Somewhat Disagree	Neither Agree nor Disagree	Somewhat Agree	Agree	Strongly Agree
The visualization exposes individual data cases and their attributes.	☐	☐	☐	☐	☐	☐	☐
The visualization facilitates perceiving relationships in the data like patterns & distributions of the variables	☐	☐	☐	☐	☐	☐	☐
The visualization promotes exploring relationships between individual data cases as well as different groupings of data cases	☐	☐	☐	☐	☐	☐	☐
The visualization helps generate data-driven questions	☐	☐	☐	☐	☐	☐	☐
The visualization helps identify unusual or unexpected, yet valid, data characteristics or values	☐	☐	☐	☐	☐	☐	☐
The visualization provides useful interactive capabilities to help investigate the data in multiple ways	☐	☐	☐	☐	☐	☐	☐
The visualization shows multiple perspectives about the data	☐	☐	☐	☐	☐	☐	☐
The visualization uses an effective representation of the data that shows related and partially related data cases	☐	☐	☐	☐	☐	☐	☐

B2. Time

How would you rate your agreement with the following statements?

	Strongly Disagree	Disagree	Somewhat Disagree	Neither Agree nor Disagree	Somewhat Agree	Agree	Strongly Agree
The visualization provides a meaningful spatial organization of the data	☐	☐	☐	☐	☐	☐	☐
The visualization shows key characteristics of the data at a glance	☐	☐	☐	☐	☐	☐	☐
The interface supports using different attributes of the data to reorganize the visualization's appearance	☐	☐	☐	☐	☐	☐	☐
The visualization supports smooth transitions between different levels of detail in viewing the data	☐	☐	☐	☐	☐	☐	☐
The visualization avoids complex commands and textual queries by providing direct interaction with the data representation	☐	☐	☐	☐	☐	☐	☐

B3. Essence

How would you rate your agreement with the following statements?

	Strongly Disagree	Disagree	Somewhat Disagree	Neither Agree nor Disagree	Somewhat Agree	Agree	Strongly Agree
The visualization provides a comprehensive and accessible overview of the data	☐	☐	☐	☐	☐	☐	☐
The visualization presents the data by providing a meaningful visual schema	☐	☐	☐	☐	☐	☐	☐
The visualization facilitates generalizations and extrapolations of patterns and conclusions	☐	☐	☐	☐	☐	☐	☐
The visualization helps understand how variables relate in order to accomplish different analytic tasks	☐	☐	☐	☐	☐	☐	☐

B4. Confidence

How would you rate your agreement with the following statements?

	Strongly Disagree	Disagree	Somewhat Disagree	Neither Agree nor Disagree	Somewhat Agree	Agree	Strongly Agree
The visualization uses meaningful and accurate visual encodings to represent the data	☐	☐	☐	☐	☐	☐	☐
The visualization avoids using misleading representations	☐	☐	☐	☐	☐	☐	☐
The visualization promotes understanding data domain characteristics beyond the individual data cases and attributes	☐	☐	☐	☐	☐	☐	☐
If there were data issues like unexpected, duplicate, missing, or invalid data, the visualization would highlight those issues	☐	☐	☐	☐	☐	☐	☐

B5. Do you have additional comments on the interactive visual analysis approach?

Section C: Glyph View

The initial loading may take some time. The application is loading as long as you browser tab states "Updating...". You can freely explore the visualization. Then, please answer the following questions based on the visual analysis approach.

Please access the visualization here: https://uncertainty-cycles.cg.informatik.uni-siegen.de/dash/?view=spatial (if you do not see a single glyph view in the top-left, please try Firefox or Safari as a Browser)

If you would like to access the introduction again, you find the information here: https://uni-siegen.sciebo.de/s/tMKnoqqiTnqAY3S

Terminology:

data case- refers to an instance of the data set; synonymous with data item or data point

attribute- refers to properties of the data cases in the data set; synonymous with feature, dimension, or variable

relationship in the data- refers to attributes among the data, such as correlations, clusters, or distributions

C1. Insight

How would you rate your agreement with the following statements?

	Strongly Disagree	Disagree	Somewhat Disagree	Neither Agree nor Disagree	Somewhat Agree	Agree	Strongly Agree
The visualization exposes individual data cases and their attributes.	☐	☐	☐	☐	☐	☐	☐
The visualization facilitates perceiving relationships in the data like patterns & distributions of the variables	☐	☐	☐	☐	☐	☐	☐
The visualization promotes exploring relationships between individual data cases as well as different groupings of data cases	☐	☐	☐	☐	☐	☐	☐
The visualization helps generate data-driven questions	☐	☐	☐	☐	☐	☐	☐
The visualization helps identify unusual or unexpected, yet valid, data characteristics or values	☐	☐	☐	☐	☐	☐	☐
The visualization provides useful interactive capabilities to help investigate the data in multiple ways	☐	☐	☐	☐	☐	☐	☐
The visualization shows multiple perspectives about the data	☐	☐	☐	☐	☐	☐	☐
The visualization uses an effective representation of the data that shows related and partially related data cases	☐	☐	☐	☐	☐	☐	☐

C2. Time

How would you rate your agreement with the following statements?

	Strongly Disagree	Disagree	Somewhat Disagree	Neither Agree nor Disagree	Somewhat Agree	Agree	Strongly Agree
The visualization provides a meaningful spatial organization of the data	☐	☐	☐	☐	☐	☐	☐
The visualization shows key characteristics of the data at a glance	☐	☐	☐	☐	☐	☐	☐
The interface supports using different attributes of the data to reorganize the visualization's appearance	☐	☐	☐	☐	☐	☐	☐
The visualization supports smooth transitions between different levels of detail in viewing the data	☐	☐	☐	☐	☐	☐	☐
The visualization avoids complex commands and textual queries by providing direct interaction with the data representation	☐	☐	☐	☐	☐	☐	☐

LimeSurvey

C3. Essence

How would you rate your agreement with the following statements?

	Strongly Disagree	Disagree	Somewhat Disagree	Neither Agree nor Disagree	Somewhat Agree	Agree	Strongly Agree
The visualization provides a comprehensive and accessible overview of the data	☐	☐	☐	☐	☐	☐	☐
The visualization presents the data by providing a meaningful visual schema	☐	☐	☐	☐	☐	☐	☐
The visualization facilitates generalizations and extrapolations of patterns and conclusions	☐	☐	☐	☐	☐	☐	☐
The visualization helps understand how variables relate in order to accomplish different analytic tasks	☐	☐	☐	☐	☐	☐	☐

C4. Confidence

How would you rate your agreement with the following statements?

	Strongly Disagree	Disagree	Somewhat Disagree	Neither Agree nor Disagree	Somewhat Agree	Agree	Strongly Agree
The visualization uses meaningful and accurate visual encodings to represent the data	☐	☐	☐	☐	☐	☐	☐
The visualization avoids using misleading representations	☐	☐	☐	☐	☐	☐	☐
The visualization promotes understanding data domain characteristics beyond the individual data cases and attributes	☐	☐	☐	☐	☐	☐	☐
If there were data issues like unexpected, duplicate, missing, or invalid data, the visualization would highlight those issues	☐	☐	☐	☐	☐	☐	☐

C5. Do you have additional comments on the interactive visual analysis approach?

D1.

Consider the following tasks:

T1: Identifying reliable structures: The goal is to find information with a high probability of occurring in a measurement. Structures that only rarely occur should be visualized while also communicating the uncertainty, such that the probability is conveyed.

T2: Visualizing different scales: The main goal of the experiment is to obtain an understanding of how structures on the microscale, such as forces between particles, relate to information on the macroscale, such as the number of nonparticipating nodes in the force network.

T3: Investigate the behavior when applying pressure: With increasing pressure, the packing fraction of the system increases. To better understand phase transitions in granular materials, the behavior of the system and associated uncertainty should be visualized as the packing fraction changes

Which of the two designs to you find more suitable to address these tasks?

Design with glyph visualization ☐
Design with small multiples ☐
They are on the same level. ☐

D2. How many years of experience do you have in visualization?

D3. Do you have experience in related application domains (granular materials, porous media, …)?

Yes ☐
A little ☐
No ☐

D4. Do you have experience in uncertainty visualization?

Yes ☐
A little ☐
No ☐

D5. What is your current role?

PhD Student ☐

Postdoc ☐

Professor ☐

Other ☐

Other

Thank you for participating!